Exploring EngageNY Strategies and Models

Starting Strong 2016

Session B12

1:00-3:00 p.m.

Facilitated by:
Carrie Black - Capital Region ESD 113 Regional Math Coordinator
Debra Kowalkowski - Northwest ESD 101, Regional Math Coordinator

Introductions

Carrie Black - Capital Region ESD 113, Regional Math Coordinator

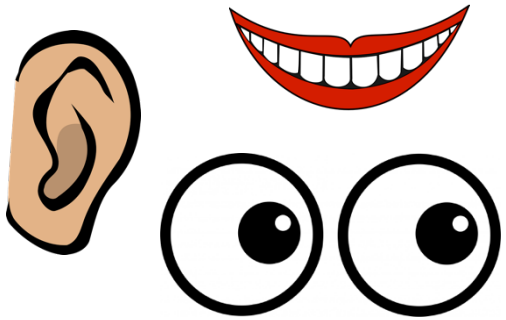
Debra Kowalkowski - Northwest ESD 101, Regional Math Coordinator

Handouts & Presentation

All slides and handouts will be available on the Starting Strong website!

www.startingstrong.net

Did you know???



- 20 percent of what they **hear**
- 30 percent of what they **see**
- 50 percent of what they **see** and **hear**
- 70 percent of what they **see**, **hear**, and **say**
  (e.g. discuss, explain to others)
- 90 percent of what they **see**, **hear**, **say**, and **do**

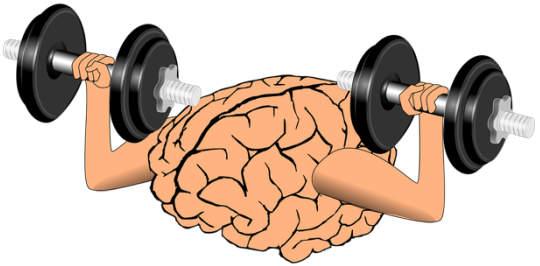
Learning Objectives:

1. Explore strategies used in lessons with EngageNY

2. Practice using the strategies

3. Collaborate with teachers using EngageNY

4. Explain how using the strategies helps build conceptual understanding



The Shifts in Math...

Focus - “...focus deeply on only the concepts that are prioritized in the standards…”

Coherence - “Principals and teachers carefully connect the learning within and across grades so that...students can build new understanding onto foundations built in previous years.”

Rigor - “...pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications.”

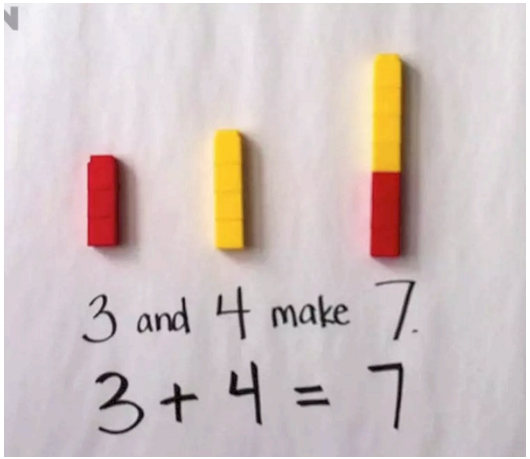
Models That Help Build Conceptual Understanding...

Number Towers
Number Path
Number Bond
Ten-Frame
Bundles
Place Value Chart
Base-Ten Blocks
Number Disks
Array and Area Model
Rekenrek
Number Line
Tape Diagram

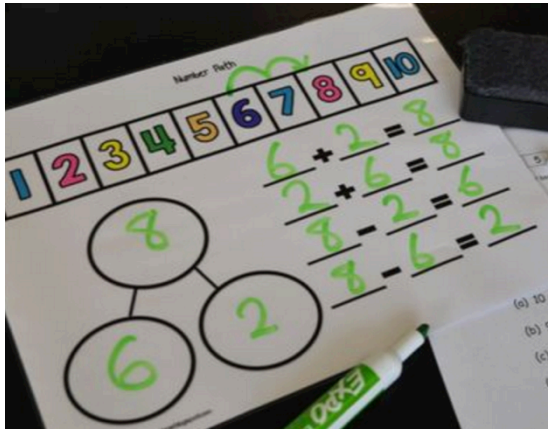
Mathematical Models in EngageNY

Numbers Through 10

Number Towers



Number Path



Number Bond

Number Towers

5 threes + 2 threes = (5 + 2) threes

3 3 3 3 3

3 3 3 3 3 3 3

Instructional Strategies with Number Towers

Sort, classify, and count up to 5 with meaning and then begin extending How Many questions up to 10.

Build a series of towers from 1 to 10, and then use the towers to relate quantities, e.g., “5 is before 6.” “6 is after 5.” “5 + 1 more is 6.” “6 is more than 5.” “6 is 1 more than 5.” “5 is 1 less than 6.” “5 and 2 make 7.” “5 + 2 = 7.”

Build a tower that shows 6.

Build a specific tower and count the cubes. (Cardinality)

Partners roll dice, each build a different tower and state which has more (less).

Instructional Strategies with Number Towers

Build a tower while stating the ‘’one more” relationship (e.g., 4, 1 more is 5). Deconstruct the tower while stating the ‘’one less” relationship (e.g., 7, one less is 6).

Count on from 5 (e.g., to count 7, students use the color change to say “5, 6, 7”instead of starting from 1). The color change at 5 may be presented to students as a shortcut by having students slide their finger over a group of 5 as they count. (Subitizing)

Count up from numbers other than 0 and 1.

Count down from numbers other than 10 to numbers other than 0 and 1.

Compare numbers within 1 and 10.

Number Paths

Instructional Strategies for Number Paths

Sort, classify, and count up to 5 with meaning and then work on extending “How Many” questions up to 10.

Match amounts to numerals.

Write numerals 1 to 5.

Extend the meaning of 6, 7, and 8 with numerals (6 is 5 and 1, 7 is 5 and 2, 8 is 5 and 3.)

Become fluent with numbers to 10 and practice “before” and “after,” as well as relationships of “1 more/less” and “2 more/less”

Order numbers from 1 to 10.

Instructional Strategies for Number Paths

Play number order games (e.g., Partner A closes eyes while Partner B covers a number with a penny— then Partner A has to guess the hidden number).

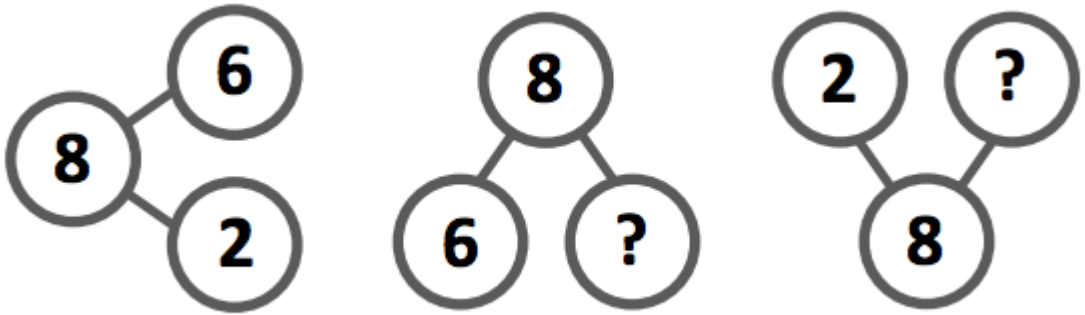
Fold the number path so that only small sections are visible. Students show 4, 5, 6, 7; teacher says “4, 5, hmm, 7 what number is missing?”

Play “I Wish I Had” games (e.g., “I wish I had 7, but I only have 5.” Student answers by placing a finger on 5 and then counting on to say “2” – the amount needed to make the target number.)

Match ordered sets with numerals on the number path.

Number Bond

Grades: Kindergarten - 5th grade



Instructional Strategies for Number Bonds

Make bonds with a specified whole using concrete objects. Students place all the objects into the “parts” circles of the bond using various combinations. These can be recorded pictorially (students draw objects in the bonds), abstractly (children write numerals in the bonds), or a combination of these representations as appropriate.

Generate number stories for each number from 5 to 10 from pictures and situations.

Develop fluency: Show all the possible ways to make ___, for all the numbers from 1 to 10.

Instructional Strategies for Number Bonds

Present bonds in which the whole and one part are visible (using concrete, pictorial, and eventually abstract representations). Students solve for the other part by bonding, counting on, or subtracting.

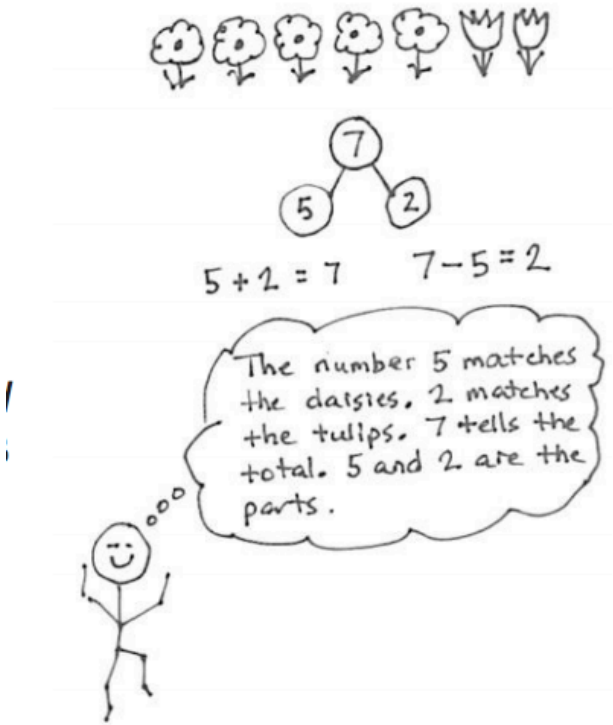
Transition students from number bonds to tape diagrams by drawing both representations for number stories.

Use number bonds as a support for mental math techniques such as “Make 10” (see grade specific examples to follow).

Use number bonds to see part-whole fraction and decimal relationships.

Number Bonds--Where they show up…

Kindergarten



Grade 1 Example

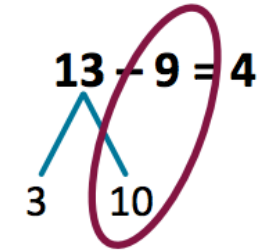
Decompose 13 into 10 and 3.

Subtract 9 from the 10.

10 – 9 = 1

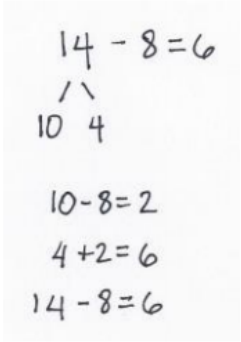
Then add 1 + 3.

1 + 3 = 4, so 13 – 9 = 4

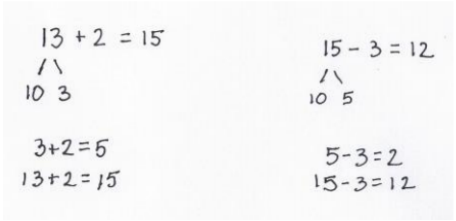


Number Bonds--Where they show up…

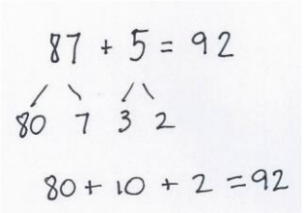
2nd Grade



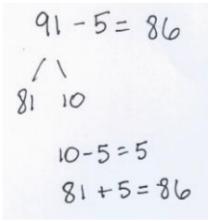
Take from 10



Add and subtract ones

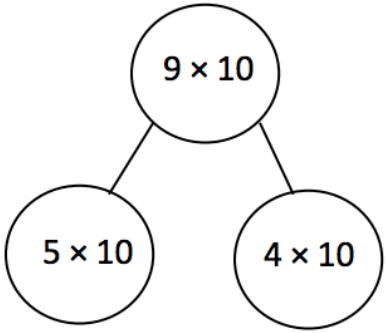


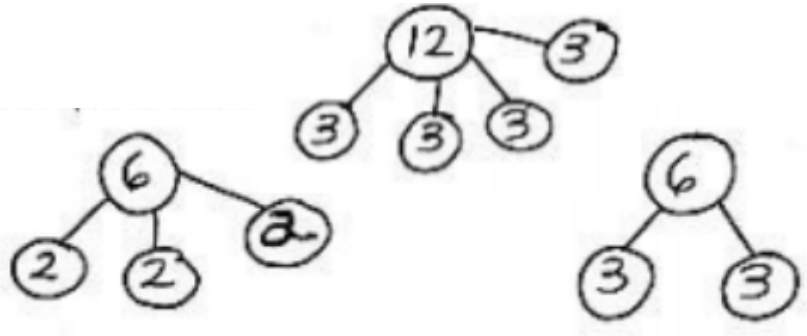
Add basic facts to cross multiples of ten



Decompose and subtract from the 10

3rd Grade





TURN AND TALK

What connects with you and the use of Number Towers, Number Paths, and Number Bonds at the elementary level?

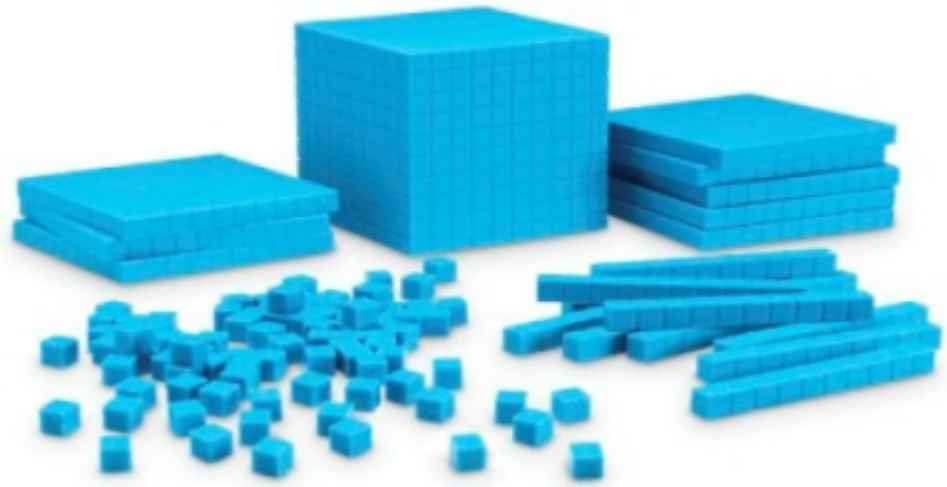
How might you use these in your classroom?

Let’s Try Something...

63 - 28

Base-Ten Blocks

Grades: Kindergarten - 2nd grade



Instructional Strategies for Base-Ten Blocks

Represent quantities on the mat and write in standard, expanded, and word form.

Play “More” and “Less” games. Begin with an amount on a mat. At a predetermined signal (e.g., teacher claps or rings a bell), students add (or subtract) a quantity (2, 5, 10, or other) to the blocks on the mat.

Give student equivalent representation riddles to be solved with base-ten pieces. For example, I have 29 ones and 2 hundreds.

What number am I?

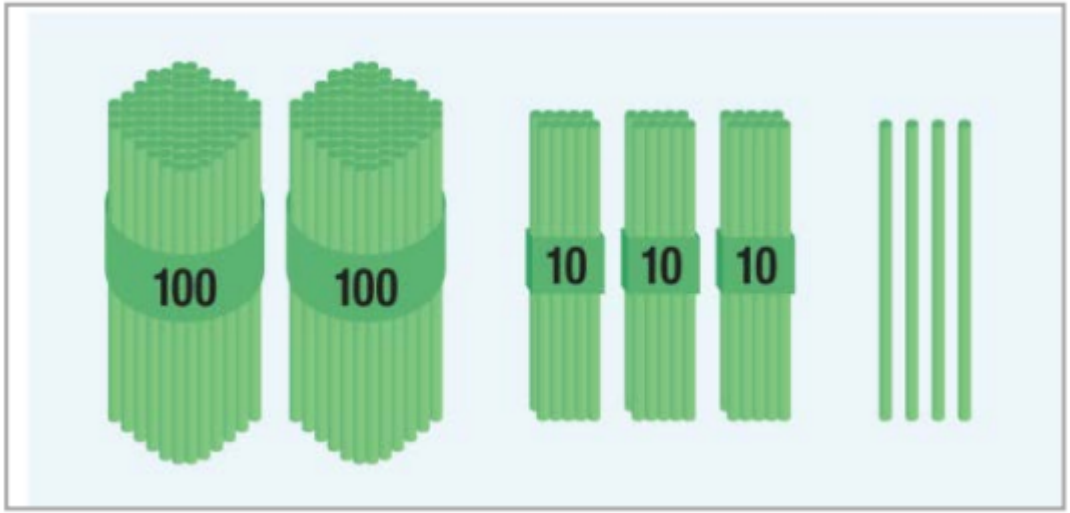
Model addition, subtraction, multiplication, and division.

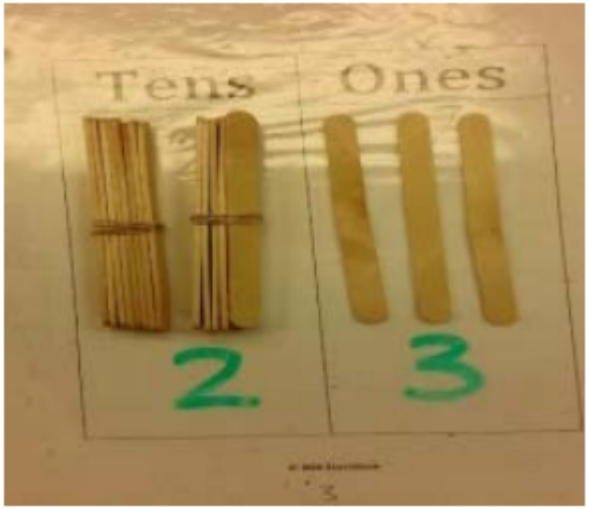
Use blocks and mats as a support for teaching students to record the standard algorithms for all four operations.

Bundles

Grades: Kindergarten - 2nd grade

Popsicle sticks and straws are great to use for these





Instructional Strategies for Bundles

Represent various quantities with bundles and “singles.”

Count school days. Each day a single straw/stick is added to the ones pocket and counted. Sticks are bundled when 10 days have passed and moved to the tens pocket. Have a “100th Day” celebration.

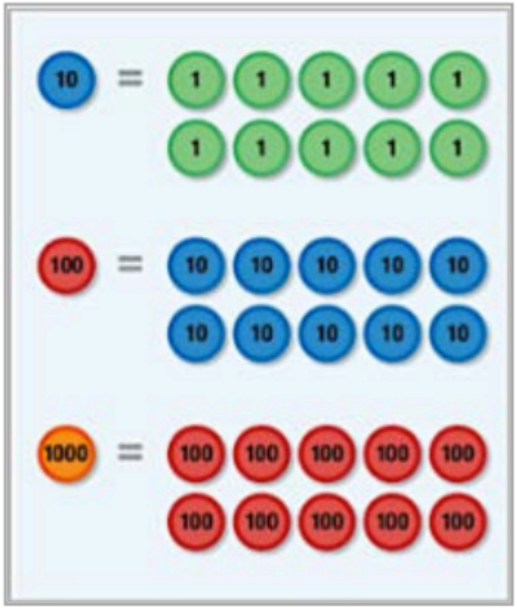
Bundles may also be used to count down to a significant event (e.g.., the last day of school), unbundling as necessary.

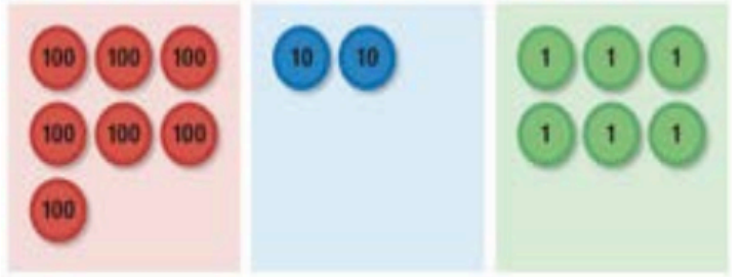
Play “Race to Zero” with a partner. Students start with a quantity between 30 and 40 in bundles. Roll two dice to determine what can be taken away from the starting quantity (unbundling as necessary). First partner to reach zero is the winner. (This game may also be played as an addition game.)

Count in unit form (2 tens, 8 ones; 2 tens, 9 ones; 3 tens, etc.).

Represent quantities on place value mats to be added or subtracted.

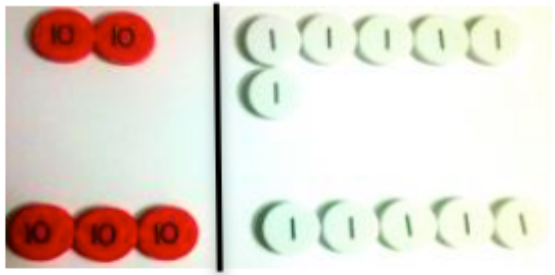
Number Disks

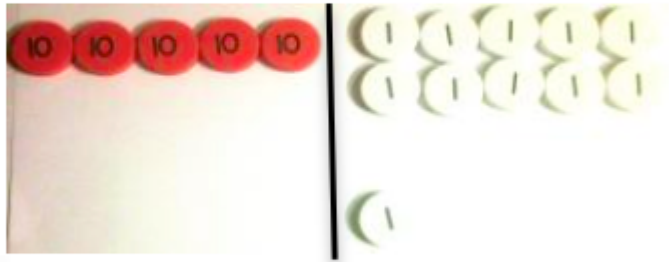




Place Value Chart with Number Disks

2nd-grade


10 10
10 10 10
1 1 1 1 1
1
1 1 1 1 1


10 10 10 10 10
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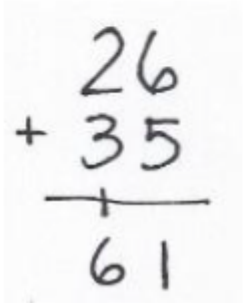
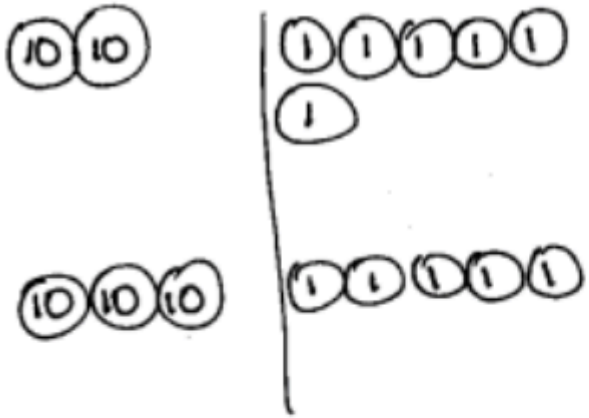
1

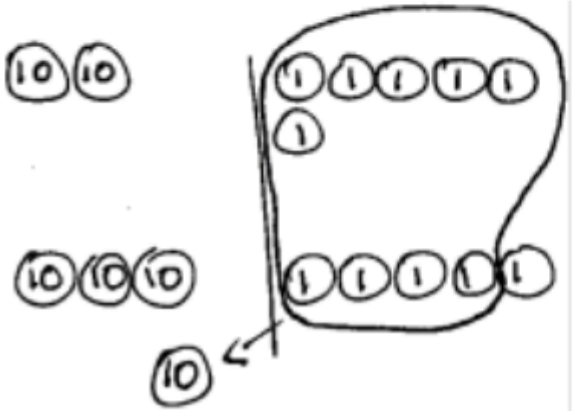

26
+ 35
61


10 10 10 10 10
10
1

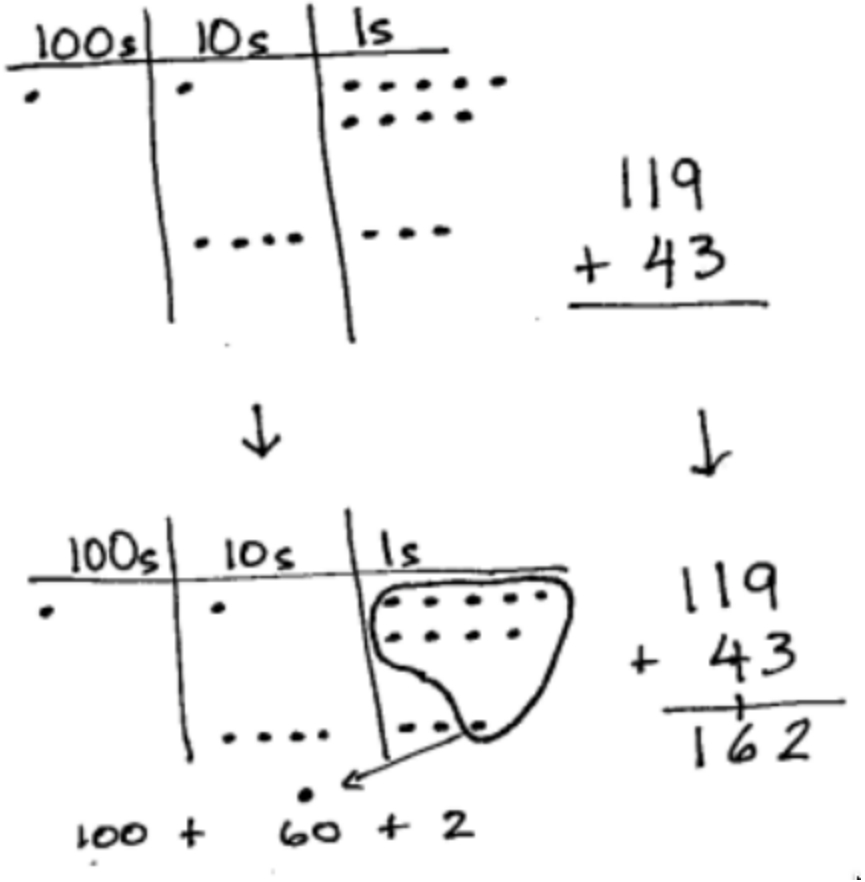
2nd-grade



$$\begin{array}{r} 26 \\ +\ 35 \\ \hline 61 \end{array}$$



2nd-grade



Instructional Strategies for Number Disks

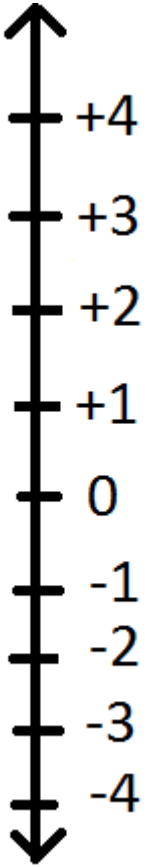
Play pattern games: “What is 100 less than 253?” Students simply remove a 100 disk and state and/or record their new number.

Play partner games: Partner A hides the disks from Partner B within a file folder. Partner A says, “I am looking at the number 241. I will make 10 less (physically removing a 10 disk). What is 10 less than 241?” Partner B writes the answer on his personal board/notebook and then states a full response: “10 less than 241 is 231.” Partner A removes the folder and the partners compare the written response with the disks.

Perform all four operations with both whole numbers and decimals on mats.

Use materials to bridge to recording the standard algorithms for all four operations with both whole numbers and decimals.

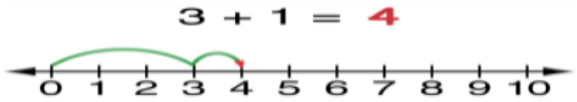
Number Lines



Instructional Strategies for Number Lines

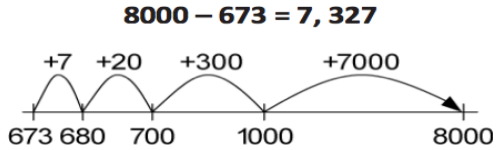
Measure lengths in meters and centimeters.

Counting on: Have students place their finger on the location for the first addend, and count on from there to add the second addend.



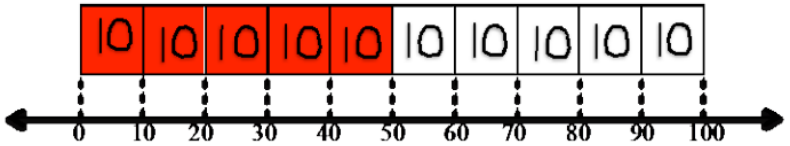
Have students use a “clock” made from a 24 inch ribbon marked off at every 2 inches to skip-count by fives.

Compute differences by counting up. 8000 – 673 = 7, 327

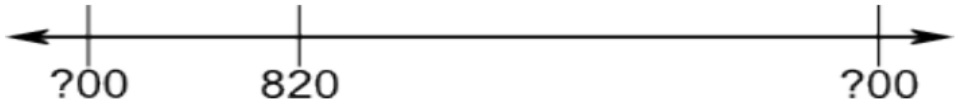


Instructional Strategies for Number Lines

Multiplying by 10; students visualize how much 5 10's is, and relate it to the number line.



Rounding to the nearest ten or hundred; e.g., students identify which 'hundreds' come before 820, "820 is between 800 and 900."



Practice with Fluency--EngageNY/Eureka Math Style

Please rise...you're going to do some activity now.

This is a free video found at
http://greatminds.net/maps/math/video-gallery/grades-k-5-fluency

Place Value Charts

Place Value Chart Without Headings
(Used with labeled materials such as disks)

Hundreds	Tens	Ones

Place Value Chart with Headings
(Used with unlabeled materials such as base-ten blocks or bundles)

Instructional Strategies for Place Value Charts

Have students build numbers on mats. Place value cards may be used to show the expanded form of a number that is represented on the place value chart.

Count the total value of ones, tens, and hundreds with any discrete, proportional or non-proportional material such as bundles, base-ten blocks or number disks.

Model and use language to tell about 1 more/less, 10 more/less on the place value chart with disks when there is change in the hundreds unit.

Instructional Strategies for Place Value Charts

Complete a pattern counting up and down.

Model addition and subtraction using base-ten blocks or number disks.

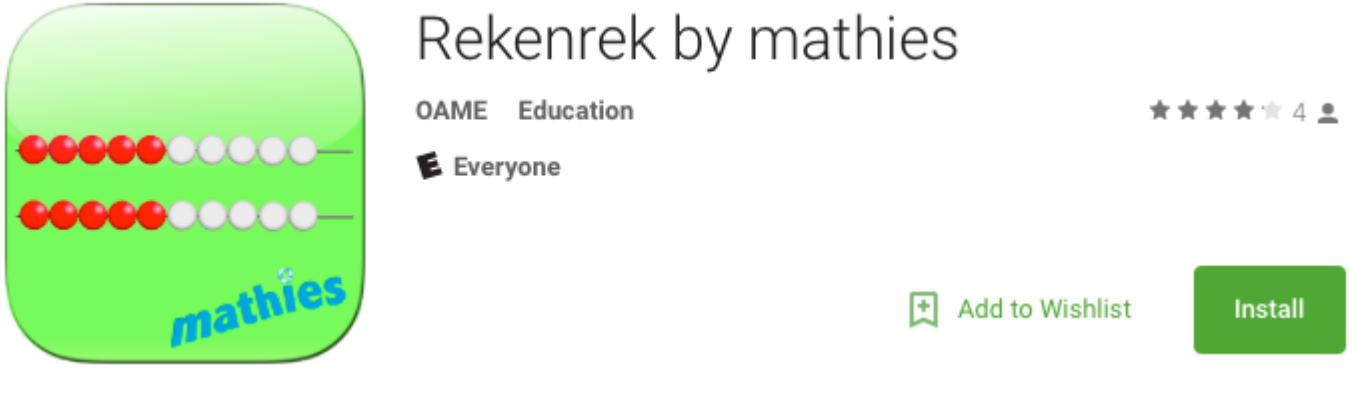
Use the mat and place value materials as a support for learning to record the standard algorithms for addition, subtraction, multiplication, and division.

Rekenrek

20-Bead Rekenrek

100-Bead Rekenrek

Rekenrek by mathies app



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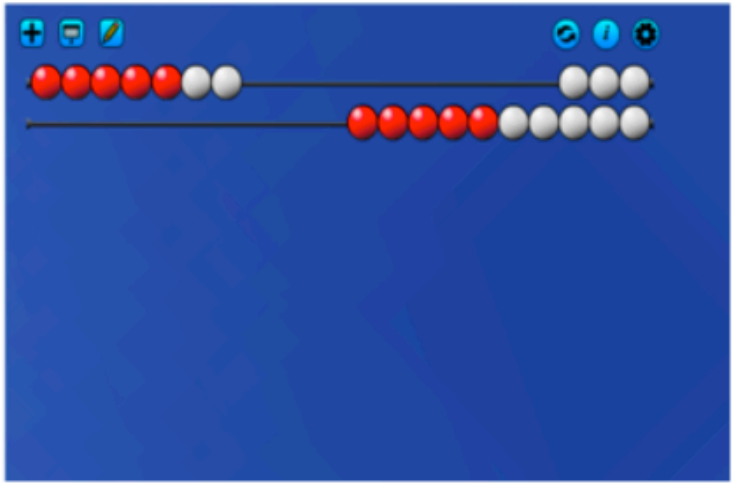
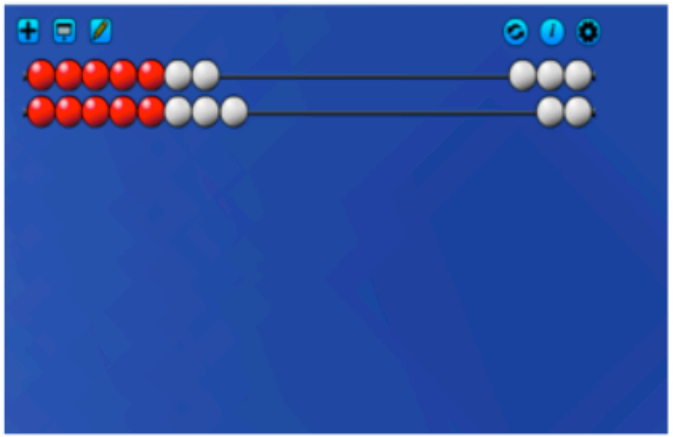


Representing

Represent numbers by sliding the beads to the left.

This shows the number 7.

Five red beads followed by two white beads helps us to see that 7 is equal to 5 + 2.



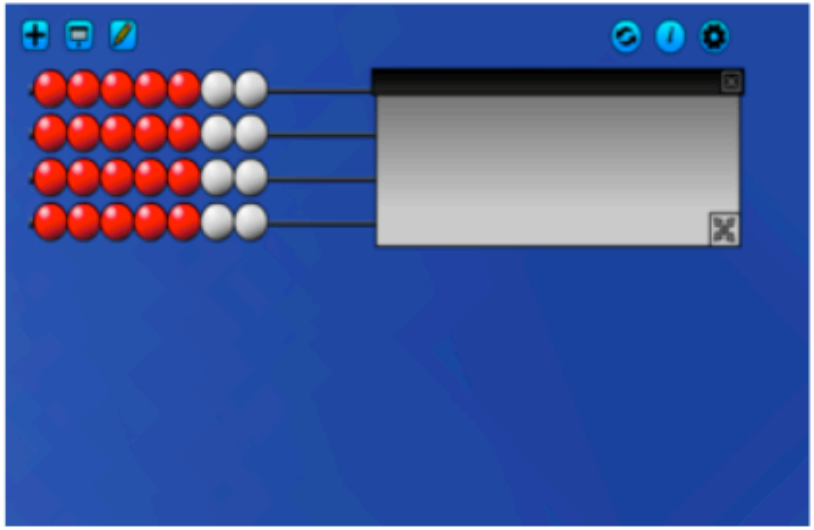
Addition and Subtraction

Slide 8 beads to the left on the second row.

This now shows 7 + 8 = 15.

Alternatively, ten red beads and five white beads help us to see that 15 is equal to 10 +5.

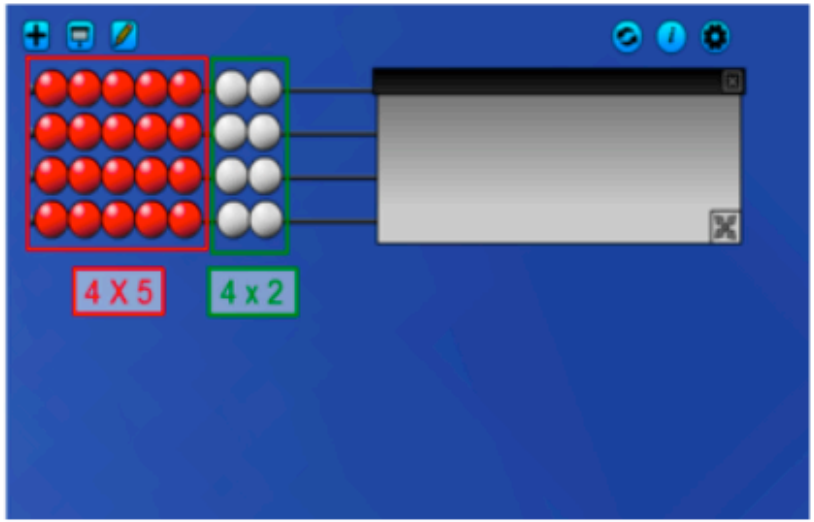
Or, if we think about the white beads on the right hand side, we can think of 15 as equal to 20 - 5.



Multiplication

This represents 4 rows of 7 or 4 x 7.

There are 28 beads in total, so we know that 4 x 7 = 28.



If we look only at the red beads, we see 4 groups of 5.

The white beads show 4 groups of 2.

So, the colours help us to see that 4 x 7 = 4 x 5 + 4 x 2.

Build your own Rekenrek

Instructional Strategies for Rekenreks

Pre-K through 1st Grade:

Count up and down in short sequences (1, 2, 3, 2, 3, 4, 3, 2,…, simulate the motion of a roller-coaster).

Think of 7 as “2 more than 5.”

See “inside” numbers (subitize – “instantly see how many”).

Count in unit form (1 ten 1, 1 ten 2, 1 ten 3… 2 tens 1, 2 tens 2, etc.).

Skip-count with complexity such as counting by 10’s on the 1’s (3, 23, 33, 43, …).

Group numbers in 5’s and 10’s. Compare Rekenrek to ten-frame.

Build fluency with doubles.

Instructional Strategies for Rekenreks

2nd Grade through 5th Grade:

Show fluency with addition and subtraction facts.

Find complements of numbers up to 10, 20, 30, …100.

Skip count by 2, 3, 4, 5, 6, 7, 8, and 9 within 100.

Identify doubles plus one and doubles minus 1.

Model rectangular arrays to build conceptual understanding of multiplication.

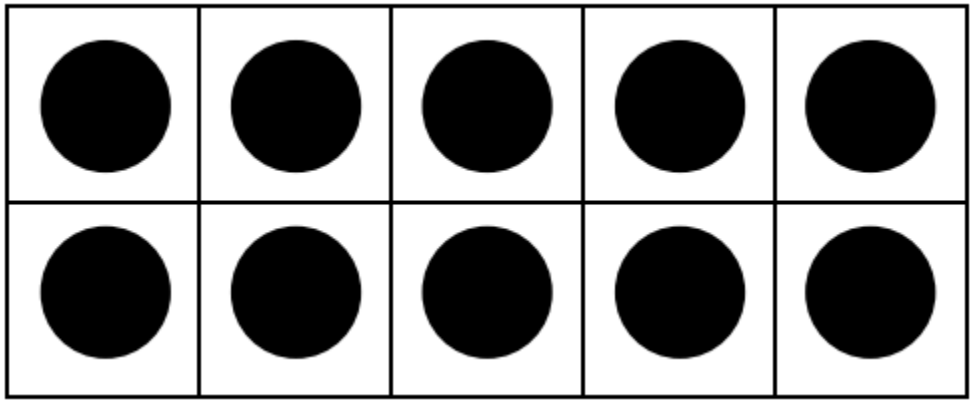
Demonstrate the distributive property. Think of 3 x 12 as 3 x 10 plus 3 x 2.

TURN AND TALK

How might you use rekenreks?
Do you already?

Do you think your students could make their own?
If not, who could you use to create a classroom set or two?

Ten-Frame



Instructional Strategies for Ten Frames

“Flash” a ten-frame for 3-5 seconds then ask students to re-create what was filled/not filled on their own personal ten-frame. (Students may also tell how many they saw or match the “flash” with a numeral card.)

Use “flash” technique, but ask students to tell 1 more or less than the number flashed.

Roll dice and build the number on the ten-frame.

Partner games: Partner 1 rolls a die and builds the number on the frame. Partner 2 rolls and adds that number to the frame (encouraging “10” and “leftovers” or using two ten-frames to represent the sum).

Instructional Strategies for Ten Frames

- Play Crazy Mixed Up Numbers. Have children represent a number on the ten-frame, then give various directions for changing the frame (e.g., start with 4 – “two more” – “one less” – “one fewer” – “double it” – “take away three”). This activity has the added benefit of providing the teacher with the opportunity to observe how students count – who clears the mat and starts over each time and who is counting on and/or subtracting.

- Write number stories about the filled and “unfilled” parts of the ten-frame.

Instructional Strategies for Ten Frames

Counting in unit form:

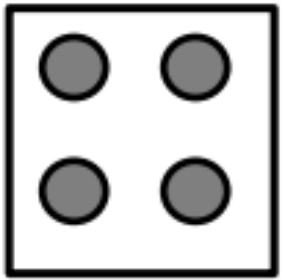
Regular	Unit Form
eleven	1 ten one
twelve	1 ten two
thirteen	1 ten three
twenty	2 tens
twenty-six	2 tens six

Represent a number between 5 and 10 on the frame with one color counter. Have students add a quantity between 6 and 9 (represented by a second color) to it (e.g., 7 + 6). Encourage students to “fill the frame” and re-state the problem as 10 + 3.

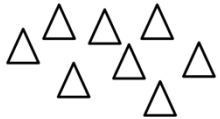
Arrays & Area Models

Kindergarten

2nd Grade

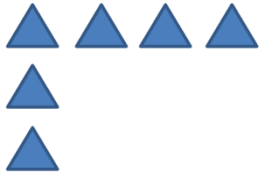


Circle groups of 4. Then draw the triangles into two equal rows.



Circle groups of 2. Redraw the groups of two as rows and then as columns.

Complete the array by drawing more triangles. The array should have 12 triangles in all.



Additional Areas Models & Arrays

3rd Grade & 4th Grade

Use the arrays below to answer each set of questions.

1. 

a. How many rows of cars are there? 4

b. How many cars are there in each row? 2

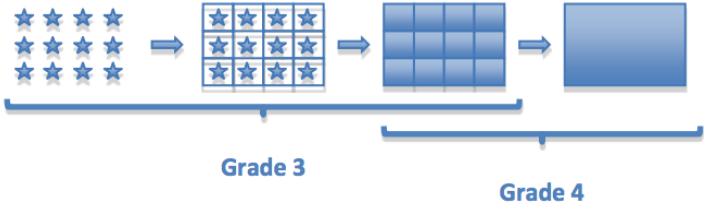
a. There are 5 rows of triangles. How many triangles are in each row? 4

b. Write a multiplication expression to describe the total number of triangles. 5x4

3. 

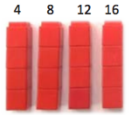
a. There are 4 spoons in each row. How many spoons are in 2 rows? 8

b. Write a multiplication expression to describe the array. 2x4



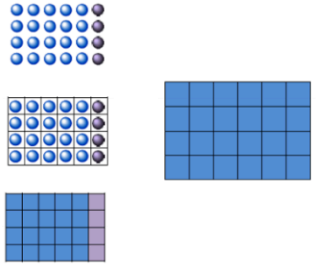
Instructional Strategies for Arrays & Area Models

Use number towers to depict multiplication problems in the shape of an array.



4 × 4 = ___.
16 ÷ 4 = __.

Use the rectangular grid to model multiplication and division.

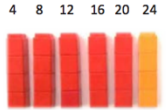


4 x 6 = ___
6 x 4 = ___

4 x ___ = 24
___ x 6 = 24

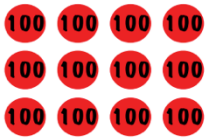
24 ÷ 4 = __
24 ÷ 6 = __

5 fours + 1 four = 6 fours
20 + 4 = 24
6 × 4 is 4 more than 5 × 4.



Multiply units with arrays.

Multiplying hundreds:
4 hundreds × 3 = 12 hundreds
400 × 3 = 1200



400 x 3 =

Area Model Activity...

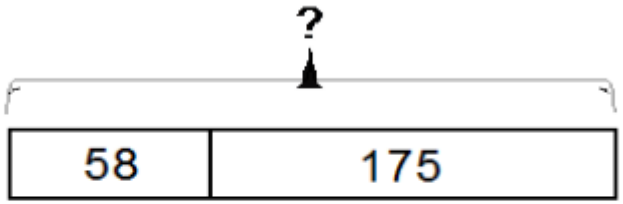
Please use an area model to represent 3 x 24

Now use an area model to represent 23 x 24

TURN AND TALK

Are you currently using area models and/or arrays?
How can arrays and area models be used at your grade level?
Discuss a problem you might use to model with an area model?

Tape Diagram

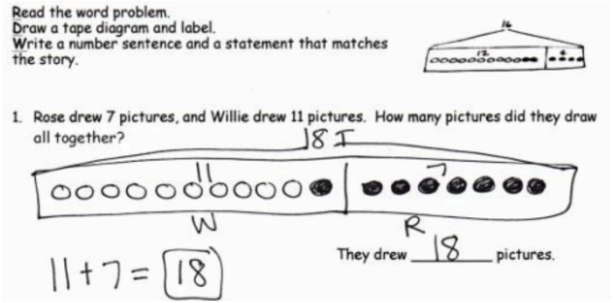


Rachel collected 58 seashells. Sam gave her 175 more. How many seashells did she have then?

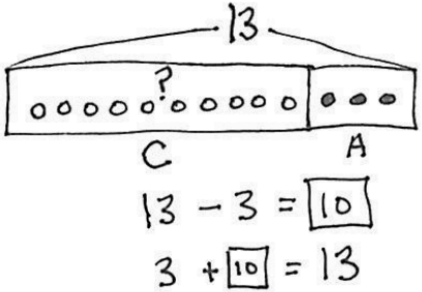
Tape Diagrams--Where they show up…

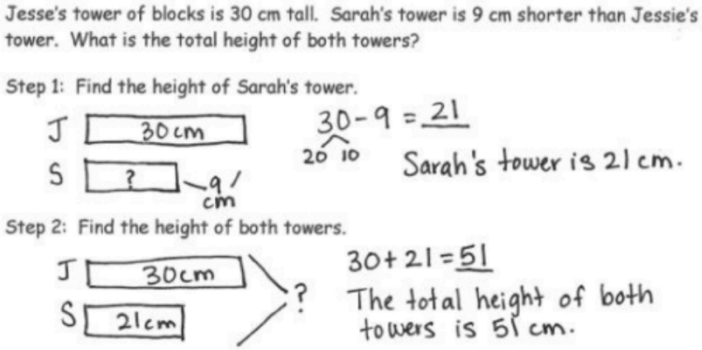
1st Grade

2nd Grade



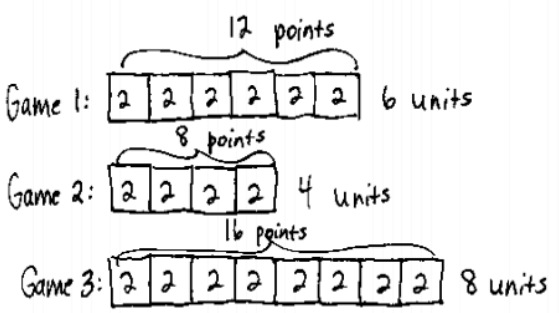
Problem 3: 13 children are on the roller coaster. 3 adults are on the roller coaster. How many people are on the roller coaster?



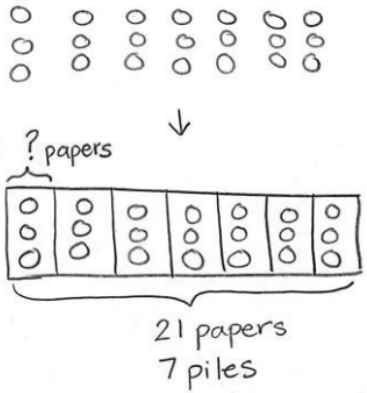


Tape Diagrams--Where they show up…

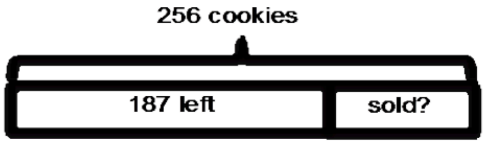
3rd Grade (3.0A.3)



Total units of 2: 6 + 4 + 8 = 18
It will take 18 total units of 2 to represent the points scored in all 3 games.



Sarah baked 256 cookies. She sold some of them. 187 were left. How many did she sell?

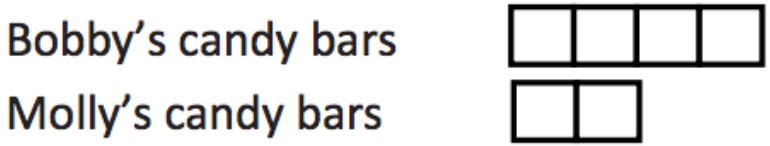


256 – 187 = ☐

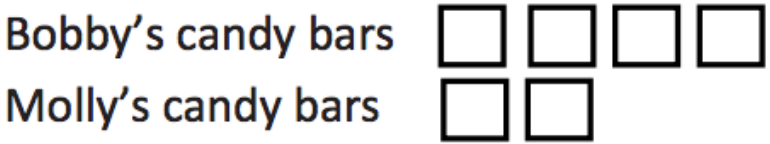
Sarah sold ☐ cookies.

Instructional Strategies for Tape Diagrams

Modeling two discrete quantities with small individual bars where each individual bar represents one unit. (This serves as an initial transition from the Unifix© cube model to a pictorial version.)

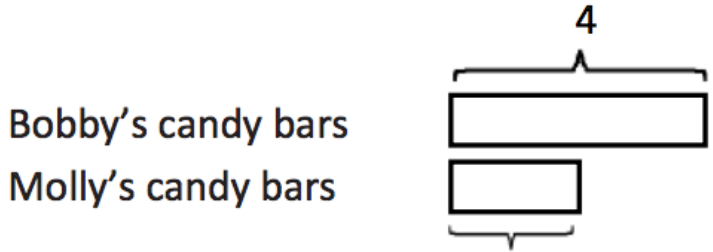


Modeling two discrete quantities with incremented bars where each increment represents one unit.



Instructional Strategies for Tape Diagrams

Modeling two quantities (discrete or continuous) with non-incremented bars.



Modeling a part-part-whole relationship where the bars represent known quantities, the total is unknown.

Modeling a part-part-whole relationship with one part unknown.

Modeling addition and subtraction comparisons.

Modeling with equal parts in multiplication and division problems.

Tape Diagram activities...

(K-1)

Sara has 5 apples. Mark brought more apples and how Sara has 9 apples. How many apples did Mark bring?

Please use a tape diagram to show this representation...

Tape Diagram activities...

(1-2)

Matteo has 5 cars, Josiah has 2 more cars than Matteo. How many cars do they have altogether?

Please use a tape diagram to show this representation...

Tape Diagram activities...

(2-3)

Sam has 7 more stamps than Joe. They have 45 stamps altogether. How many does each boy have?

Please use a tape diagram to show this representation...

Tape Diagram activities...

(3-4)

Desmond has 5 times as many toy cars as Luke. They have a total of 42 cars. How many cars does each boy have?

Please use a tape diagram to show this representation...

Tape Diagram activities...

(MS)

Two pears and a pineapple cost $2.00. Two pears and 3 pineapples cost $4.50. Find the cost of a pineapple.

Please use a tape diagram to show this representation...

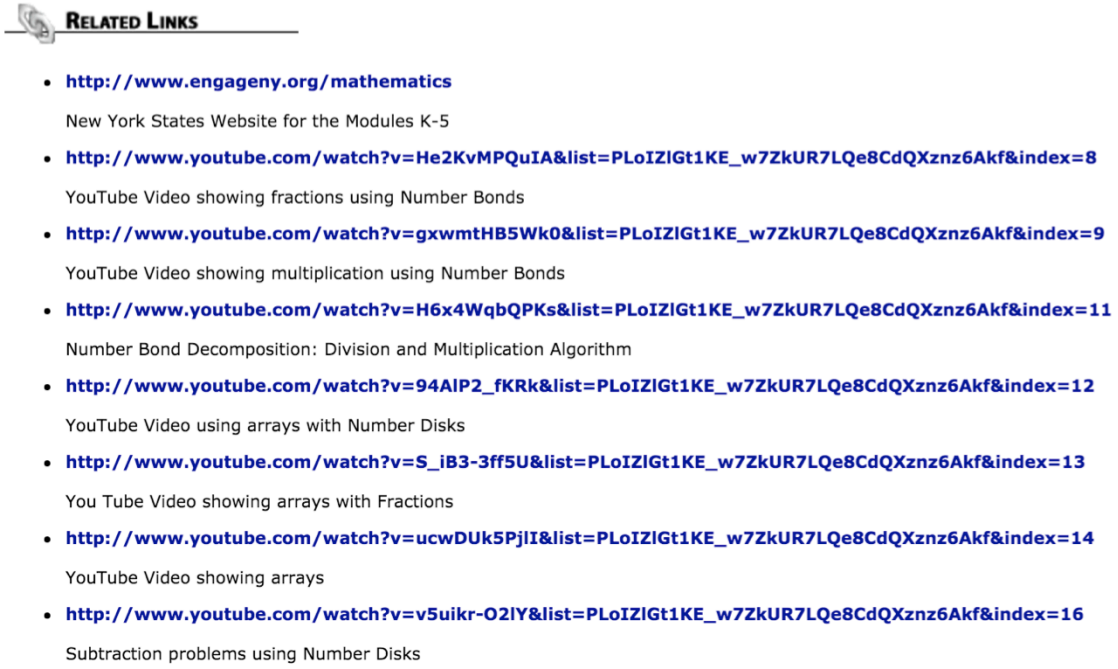
Reflection...

How does using these strategies help build conceptual understanding?

What about coherence?

Great resource of EngageNY/Eureka Math videos...

Oswego City K-5 grades for tools and their use
http://www.oswego.org/instruction.cfm?subpage=9955

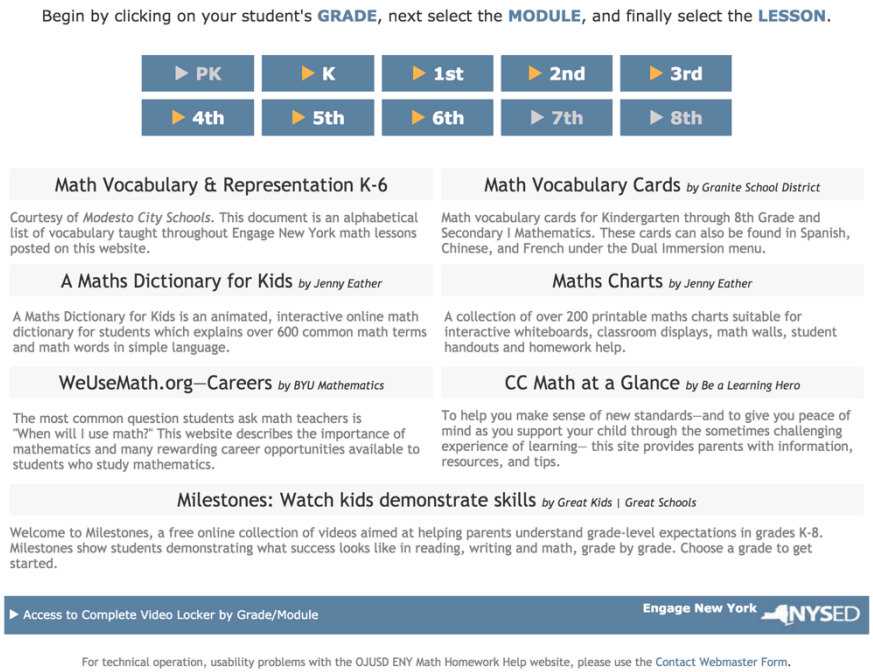


Related Links

- **http://www.engageny.org/mathematics**
  New York States Website for the Modules K-5
- **http://www.youtube.com/watch?v=He2KvMPQuIA&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=8**
  YouTube Video showing fractions using Number Bonds
- **http://www.youtube.com/watch?v=gxwmtHB5Wk0&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=9**
  YouTube Video showing multiplication using Number Bonds
- **http://www.youtube.com/watch?v=H6x4WqbQPKs&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=11**
  Number Bond Decomposition: Division and Multiplication Algorithm
- **http://www.youtube.com/watch?v=94AlP2_fKRk&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=12**
  YouTube Video using arrays with Number Disks
- **http://www.youtube.com/watch?v=S_iB3-3ff5U&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=13**
  You Tube Video showing arrays with Fractions
- **http://www.youtube.com/watch?v=ucwDUk5PjlI&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=14**
  YouTube Video showing arrays
- **http://www.youtube.com/watch?v=v5uikr-O2lY&list=PLoIZlGt1KE_w7ZkUR7LQe8CdQXznz6Akf&index=16**
  Subtraction problems using Number Disks

More great examples of EngageNY/Eureka Math...

Oakdale, CA School District - http://www.oakdale.k12.ca.us/cms/page_view?d=x&piid=&vpid=1391596408603



Resources:

Pinterest for Templates--look for quality...some fun templates out there!

Teachers Pay Teachers

*Ericabohrer.blogspot.com (free Number Path & Number Bond materials)

*Mrs. Bibeau (free number line)

www.HaveFunTeaching.com (free Place Value Charts)

http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm (free Place Value Charts)

http://teacherblogspot.com/?p=984 Teacher Blog Spot (free Number Disks)

http://greatminds.net/maps/math/video-gallery/ videos from Eureka

Before you leave...we value feedback

Thank you!

It was a pleasure working with you today. Please don’t hesitate to e-mail us with questions, suggestions, ideas, etc.

Have fun kicking off a new school year!

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dkowalkowski@esd101.net